Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

May 2023

Mathematics: applications and interpretation

Higher level

Paper 1

23 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

R Marks awarded for clear **Reasoning**.

AG Answer given in the question and so no marks are awarded.

FT Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme eg ***M1***, ***A2***.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***A3***, ***M2*** *etc.*, do **not** split the marks, unless there is a note.
- The response to a "show that" question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part. Examples:

	Correct answer seen	Further working seen	Any FT issues?	Action
1.	$8\sqrt{2}$	$5.65685...$ *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	$0.468111\ldots$ (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g.** ***(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version.
- If the error leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.
- If the candidate’s answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This includes when candidates fail to complete a “show that” question correctly, and then in subsequent parts use their incorrect answer rather than the given value.
- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.
- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was “Hence”.

5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.
- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.
- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and $1{,}9$ or 1000 and $1{,}000$ and 1.000.
- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures*.

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer to a "correct" level of accuracy (e.g 3 sf) in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^{x}$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

A GDC is required for this paper, but If you see work that suggests a candidate has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

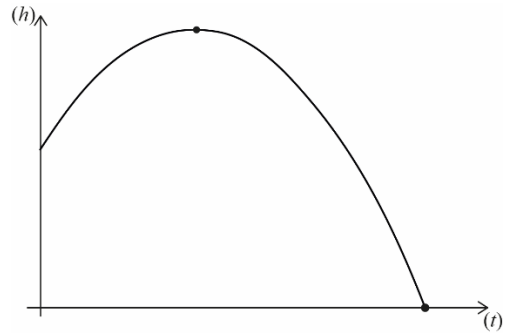
10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is "first".

1. (a) **METHOD 1**

correct sketch with some indication of maximum point ***(M1)***



0.921 (seconds) $\left(0.921052..., \frac{35}{38}\right)$ ***A1***

METHOD 2

correct substitution into equation for line of symmetry ***(M1)***

$(t =) \ -\frac{8.75}{2\times -4.75}$

0.921 (seconds) $\left(0.921052..., \frac{35}{38}\right)$ ***A1***

METHOD 3

equating the correct derivative to 0 ***(M1)***

$-9.5t + 8.75 = 0$

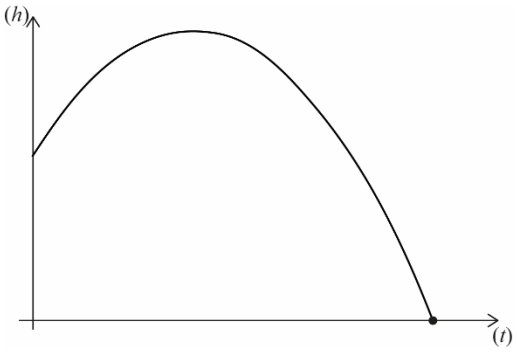
0.921 (seconds) $\left(0.921052..., \frac{35}{38}\right)$ ***A1***

Note: Award ***M1A0*** for a final answer of 0.92 seen with no working.

[2 marks]

(b) **METHOD 1**

correct sketch with some indication of x-intercept



(M1)

Note: May be seen in part (a).

2 (seconds) ***A1***

continued…

Question 1 continued

METHOD 2
setting the equation to zero ***(M1)***

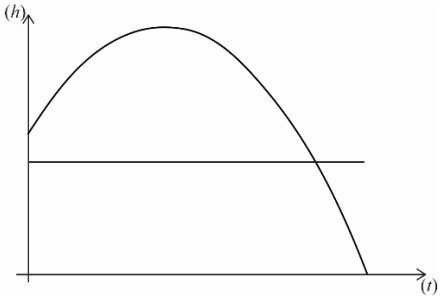
$-4.75t^2 + 8.75t + 1.5 = 0$

2 (seconds) ***A1***

Note: If both roots are given, with or without working, award ***(M1)A0***.

[2 marks]

(c) **METHOD 1**
correct sketch of quadratic function and a straight line in approximate correct position ***(M1)***



1.88 (seconds) $(1.87577\ldots$(seconds)) ***A1***

METHOD 2
setting the equation equal to 1.2 ***(M1)***
$-4.75t^2 + 8.75t + 1.5 = 1.2$
1.88 (seconds) $(1.87577\ldots$(seconds)) ***A1***

Note: Award ***(M1)A0*** if $-0.0336702\ldots$ seen as (part of) a final answer.
Award ***M1A0*** for answer of 1.9 seen without working.

[2 marks]

(d) *Award **R1** for a sensible reason in the context of the question:* ***R1***
e.g.
The model ignores air resistance (or wind)
The model treats the ball as a point
The model assumes gravity is constant
The model assumes that the ball continues to follow the trajectory even after hitting the ground
This model ignores the bouncing back of the ball after hitting the ground

Note: Do not accept generic criticisms of any mathematical model, such as:
There are assumptions being made
Models are never accurate / It is only a model

[1 mark]

Total [7 marks]

2. (a) $11.0\ (11.0212\ldots)$ ***A2***

Note: Award ***A1*** for a final answer of 11 if no unrounded answer is seen.

[2 marks]

(b) **EITHER**

$11.0 > 9.488\ (11.0212\ldots > 9.488\ )$ ***R1***

OR

$0.0263 < 0.05\ (0.0263264\ldots < 0.05\ )$ ***R1***

THEN

EITHER

(there is significant evidence to) reject H_0 ***A1***

OR

(there is significant evidence that) the (food) quality and the type of meal are not independent ***A1***

Note: Do not award ***R0A1.***

Award ***R1*** for $\chi^2_{\text{calc}} > \chi^2_{\text{crit}}$, provided the calculated value is explicitly seen in part (b).

Accept "p-value < significance level" provided their p-value is seen and their p-value is between 0 and 1.

[2 marks]

Total [4 marks]

3. (a) attempting to use $\mathrm{P}(R \cap S) = \mathrm{P}(R)\mathrm{P}(S)$ ***(M1)***

$0.2 = 0.8(0.2 + x)$ ***(A1)***

$x = 0.05$ ***A1***

[3 marks]

(b) $x + 0.2 + 0.6 + y = 1$ ***(M1)***

$y = 0.15$ ***A1***

[2 marks]

(c) **METHOD 1**

attempting to apply $\mathrm{P}(R' \mid S') = \dfrac{\mathrm{P}(R' \cap S')}{\mathrm{P}(S')}$ ***(M1)***

$\dfrac{0.15}{0.2}$

$= \dfrac{3}{4}$ ***A1***

METHOD 2

$\mathrm{P}(R' \mid S') = \mathrm{P}(R')$ (because R, S are independent) ***(M1)***

$= 1 - 0.25 = 0.75$ ***A1***

Note: ***FT*** from their values of x or y.

[2 marks]

Total [7 marks]

4. (a) **METHOD 1 (use of financial app in GDC)**

$N = 5$	**OR**	$N = 20$
$I\% = 1.2$		$I\% = 1.2$
$PV = \pm 520$		$PV = \pm 520$
$P/Y = 1$		$P/Y = 4$
$C/Y = 4$		$C/Y = 4$

(M1)(A1)

Note: Award ***M1*** for evidence of using the financial app on the calculator, ***A1*** for all correct entries.

($) 552.11 ***A1***

Note: Award at most ***(M1)(A1)A0*** if correct answer is not given to two decimal places.

METHOD 2 (use of formula)

attempt to substitute into compound interest formula ***(M1)***

$$520 \times \left(1 + \frac{1.2}{100 \times 4}\right)^{5 \times 4} \quad \textit{(A1)}$$

($) 552.11 ***A1***

Note: Award at most ***(M1)(A1)A0*** if correct answer is not given to two decimal places.

[3 marks]

(b) **EITHER**

$N = 5$

$I\% = 43.5 \ (43.4772...(\%))$

$PV = \pm 520$

$FV = \mp 30$ ***(M1)(A1)A1***

Note: Award ***M1*** for evidence of using the finance app on the calculator, ***A1*** for all correct entries, ***A1*** for correct final answer. Condone missing -/+ sign if the correct final answer is seen.

OR

$$30 = 520\left(1 - \frac{r}{100}\right)^5 \quad \text{(or equivalent)} \quad \textit{(M1)(A1)}$$

$(r =) \ 43.5\% \ (43.477...\%)$ ***A1***

Note: Award ***M1*** for using the compound interest formula, ***A1*** for correct substitutions and for equating to 30, ***A1*** for correct final answer. Accept $(r =) \ -43.5\%$.
Award ***M1A1A0*** for a final answer of 56.5%.

[3 marks]
Total [6 marks]

5. (a) $X \sim \mathrm{N}(4, 0.25^2)$

EITHER

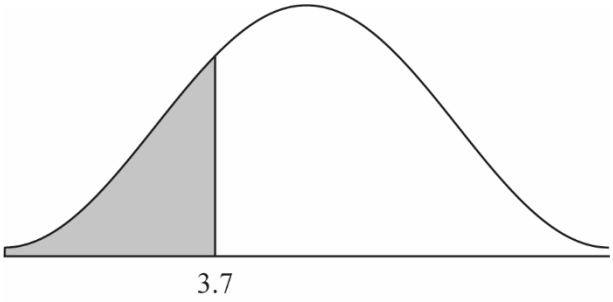
correct probability expression ***(M1)***

$\mathrm{P}(X < 3.7)$

Note: Accept a weak or strict inequality, and any label instead of X, e.g. length or L.

OR

normal curve with vertical line, left of mean, labelled 3.7, and shaded region ***(M1)***



THEN

0.115 $(0.115069\ldots, 11.5\%)$ ***A1***

Note: Award ***M1A0*** for 0.12 if no previous working.

[2 marks]

(b) **EITHER**

Correct probability expression ***(M1)***

$(\mathrm{P}(X < k) = 0.7$ **OR** $\mathrm{P}(X > k) = 0.3$

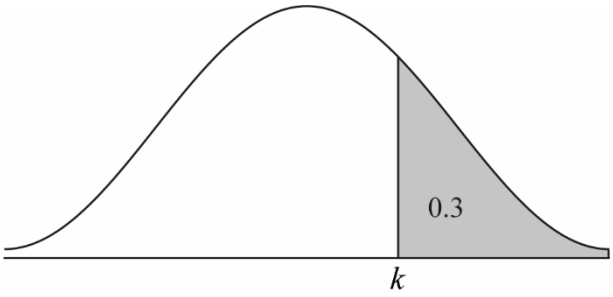
Note: Accept a weak or strict inequality, and any label instead of X *e.g.*, length or L.

continued…

Question 5 continued

OR

normal curve with vertical line to the right of the mean and shaded region, correctly labelled either 0.3 or 0.7 ***(M1)***



THEN

$(k =)\ 4.13\ (4.13110\ldots)$ ***A1***

Note: Award ***M1A0*** for 4.1 if no previous working.

[2 marks]

(c) **EITHER**

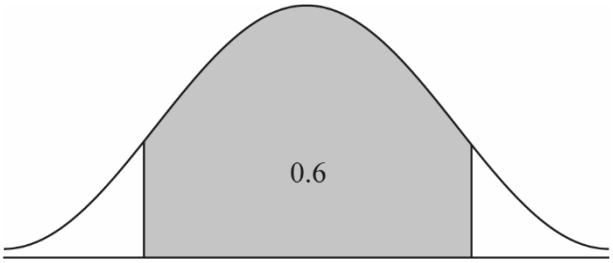
correct probability equation ***(M1)***

$\mathrm{P}(\text{length} < 4 + m) = 0.8$ **OR** $\mathrm{P}(\text{length} < 4 - m) = 0.2$

Note: Accept any letter instead of "length" *e.g.*, X or L.

OR

normal curve with vertical lines symmetrical about the mean line with a correct indication of an area of 0.6 or 0.2 or 0.8 ***(M1)***



THEN

$0.210\ (0.210405\ldots)$ ***A1***

Note: Award ***(M1)A0*** for an answer of 3.7895 or 4.2105 seen without working. Condone 0.21 seen and award ***(M1)A1***.

[2 marks]

Total [6 marks]

6. (a) **EITHER**

$\frac{4}{3}\pi(3.4)^3$ ***(A1)***

Multiplying their volume by $\frac{4}{5}$ ***(M1)***

OR

$\frac{4}{3}\pi(3.4)^3$ ***(A1)***

Subtracting $\frac{1}{5}$ of their volume ***(M1)***

$\left(\frac{4}{3}\pi(3.4)^3 - \frac{1}{5}\times\frac{4}{3}\pi(3.4)^3\right)$

Note: The ***M1*** can be awarded for a final answer of $32.9272\ldots$ seen without working.

THEN

$132\ \text{cm}^3\ \ (131.708\ldots\ \text{cm}^3)$ ***A1***

[3 marks]

(b) $\pi\times 3\times 11$ ***(A1)***

$103.672\ldots\ (\text{cm}^2)$ **OR** $33\pi\ (\text{cm}^2)$

$104\ (\text{cm}^2)$ ***A1***

[2 marks]

Total [5 marks]

7. (a) $(56\times 0.86) = 48.2\ \ (48.16)$ ***A1***

Note: Accept 48.

[1 mark]

(b) recognizing binomial distribution (may be seen in (a)) ***(M1)***

e.g. $X \sim \text{B}(56,\ 0.86)$

$\left(\text{P}(X \geq 50) =\right)\ 0.316$ ***A2***

[3 marks]

(c) $\text{P}(X \leq n) \geq 0.25$

$n = 46$ ***A2***

[2 marks]

Total [6 marks]

8. (a) attempt to create a 5x5 adjacency matrix ***(M1)***

$$\boldsymbol{M} = \begin{pmatrix} 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix} \quad \textbf{A2}$$

Note: Allow the transposed matrix. Award ***A2*** for all entries correct, ***A1*** if one or two entries are incorrect, ***A0*** otherwise.
Answer presented in markscheme assumes ABCDE ordering of rows and columns; accept other orders provided they are clearly communicated.
Award ***A1*** if the zeroes are replaced by blank cells.

[3 marks]

(b) (i) recognizing need to find $\boldsymbol{M}^7$ ***(M1)***

$$\boldsymbol{M}^7 = \begin{pmatrix} 8 & 8 & 17 & 8 & 13 \\ 8 & 10 & 19 & 17 & 14 \\ 6 & 11 & 16 & 10 & 17 \\ 11 & 8 & 19 & 14 & 10 \\ 2 & 6 & 8 & 11 & 8 \end{pmatrix}$$

2 (routes) ***A1***

(ii) vertices visited in order are
EITHER
$E \to D \to C \to B \to C \to B \to D \to A$ ***A2***

OR
$E \to D \to C \to B \to C \to E \to D \to A$ ***A2***

[4 marks]
Total [7 marks]

9. (a)

Athlete	A	B	C	D	E	F	G	H
Age rank	7	6	3	5	4	2	8	1
Time rank	3.5	2	3.5	6	7	8	1	5

A1A1

Note: Award ***A1*** for each correct row.

[2 marks]

(b) $r_s = -0.671\ (-0.670670...)$ ***A2***

Note: Only follow through from an incorrect table provided the ranks are all between 1 and 8.
Award ***A1*** for -0.67 **OR** for the omission of the negative sign, e.g. 0.671 $(0.670670...)$ or 0.67

[2 marks]

(c) (A value of $r_S = -0.671$) indicates a negative correlation between a person's age and the best time they take to run 100m. ***R1***

Note: Condone any comment that includes "weak" or "strong" etc. Accept an interpretation in words, but only if there is a general link described and not a rule: "The older a person gets, the faster they *tend to* run". Answer must be in context.

[1 mark]

(d) *Award **R1** for any sensible reason:* ***R1***
The correlation, such that it is, is unlikely to be linear for this type of data.
Spearman's CC is less sensitive to outliers
Sung-Jin is not sure the data is drawn from a bivariate normal distribution
There are outliers/extreme data
Same time for two athletes with significantly different ages

[1 mark]

(e) (i) $0.264\ (0.263762...)$ ***A2***

Note: Award ***A1*** for 0.26 with no working. Given that the exact model is not specific in the question, accept correct r^2 values from other regression models: 0.631, 0.650, 0.759 and 0.256.

(ii) approximately 26% of the variability in the times taken can be explained by the runner's age. ***R1***

[3 marks]
Total [9 marks]

10. (a) H_0 : there is no particular preference for any of the flavours ***A1***

H_1 : there is a particular preference for some of the flavours ***A1***

Note: Accept equivalent statements such as "H_0 : the population ratio of flavour preferences is $1:1:1:1$" or "H_0 : the population proportions are equal for each flavour" or "H_0 : the data is drawn from a uniform distribution".

[2 marks]

(b) **EITHER**

$p\text{-value} = 0.0629 \ \ (0.0629034...)$ ***A2***

$0.0629 > 0.05$ ***R1***

OR

$\chi^2_{\text{calc}} = 7.30$ ***A2***

$7.30 < 7.82$ ***R1***

Note: Award ***A2*** for either $p\text{-value} = 0.063$ or $\chi^2_{\text{calc}} = 7.3$ seen. Award ***R1*** for a correct comparison involve their $p\text{-value}$ or χ^2_{calc}, and follow through for their conclusion.

THEN

so there is insufficient evidence to reject H_0, i.e.

there is no particular preference for any of the flavours. ***A1***

Note: Do not award ***R0A1***.

[4 marks]

Total [6 marks]

11. (a) $30\sin(t+60^\circ)+60\sin(t+10^\circ)$ ***(M1)***

finding maximum graphically

$82.5 \text{ (V)} \ \ (82.5471...)$ ***A1***

Note: Award ***M1A0*** for 83.

[2 marks]

(b) recognizing that a is still 1 ***A1***

$V_0 = 82.5$ ***A1***

attempt to find an x-intercept of combined voltage ***(M1)***

$b = 26.2^\circ \ \ (26.1643...^\circ)$ **OR** any other correct x-intercept ***A1***

Note: May be seen in the final answer. Award ***M1A0*** for $b = 26$ with no working.

$\left(V_{\text{TOT}} = 82.5\sin(t+26.2^\circ) \ \ \left(82.5471...\sin(t+26.1643...^\circ)\right)\right)$

Note: Award at most ***(M1)A1(A1)A0*** if phase shift of $-153.835\ldots$ is seen in the final answer. In part (b), candidates may use $\arg\left(30e^{60i}+60e^{10i}\right)$ to determine the new phase shift, and hence could be awarded ***M1*** for this valid method.

[4 marks]

Total [6 marks]

12. (a) equating volume of sphere formula to 288π ***(M1)***

$$\frac{4}{3}\pi r^3 = 288\pi$$

$$\Rightarrow r = 6 \text{ (cm)}$$ ***A1***

[2 marks]

(b) $\frac{\mathrm{d}V}{\mathrm{d}r} = 4\pi r^2$ (seen anywhere) ***(A1)***

$$\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{\mathrm{d}V}{\mathrm{d}r}\frac{\mathrm{d}r}{\mathrm{d}t}$$ ***(M1)***

$$\frac{\mathrm{d}V}{\mathrm{d}t} = 4\pi r^2 \frac{\mathrm{d}r}{\mathrm{d}t}$$

$$15 = 4\pi \times 6^2 \times \frac{\mathrm{d}r}{\mathrm{d}t}$$

$$\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{15}{144\pi}(\text{cm s}^{-1}) \ (0.0332,\ 0.0331572...)$$ ***A1***

[3 marks]

Total [5 marks]

13. (a) **METHOD 1 (find product of matrices first)**

$T \to T'$ is represented by $\boldsymbol{QP} = \begin{pmatrix} -4 & 1 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} 3 & 1 \\ 0 & 2 \end{pmatrix}$ ***(M1)***

$= \begin{pmatrix} -12 & -2 \\ 3 & 7 \end{pmatrix}$ ***(A1)***

recognizing need to find their $(\boldsymbol{QP})^{-1}$ ***(M1)***

$(\boldsymbol{QP})^{-1} = \begin{pmatrix} -12 & -2 \\ 3 & 7 \end{pmatrix}^{-1}$

$= -\frac{1}{78}\begin{pmatrix} 7 & 2 \\ -3 & -12 \end{pmatrix}$ **OR** $= \begin{pmatrix} -0.0897435... & -0.0256410... \\ 0.0384615... & 0.153846... \end{pmatrix}$ ***A1***

METHOD 2 (find inverses of both matrices first)

recognizing need to find inverse of both $\boldsymbol{P}$ and $\boldsymbol{Q}$ ***(M1)***

$\boldsymbol{P}^{-1} = \begin{pmatrix} \frac{1}{3} & -\frac{1}{6} \\ 0 & \frac{1}{2} \end{pmatrix}$ **AND** $\boldsymbol{Q}^{-1} = \begin{pmatrix} -\frac{3}{13} & \frac{1}{13} \\ \frac{1}{13} & \frac{4}{13} \end{pmatrix}$ ***(A1)***

$T' \to T$ is represented by $\boldsymbol{P}^{-1}\boldsymbol{Q}^{-1} = \begin{pmatrix} 3 & 1 \\ 0 & 2 \end{pmatrix}^{-1}\begin{pmatrix} -4 & 1 \\ 1 & 3 \end{pmatrix}^{-1}$ ***(M1)***

$= -\frac{1}{78}\begin{pmatrix} 7 & 2 \\ -3 & -12 \end{pmatrix}$ **OR** $= \begin{pmatrix} -0.0897435... & -0.0256410... \\ 0.0384615... & 0.153846... \end{pmatrix}$ ***A1***

Note: In METHOD 1, award ***M1A0M1A0*** if they multiply the matrices in the wrong order.
In METHOD 2, award ***M1A1M1A0*** if they multiply the matrices in the wrong order.

[4 marks]

(b) $\left(\det\left[-\frac{1}{78}\begin{pmatrix} 7 & 2 \\ -3 & -12 \end{pmatrix}\right] =\right) -\frac{1}{78}$ **OR** $\left(\det\begin{pmatrix} -12 & -2 \\ 3 & 7 \end{pmatrix} =\right) -78$ ***(A1)***

area of $T' = |\det \boldsymbol{QP}| \times$ area of T **OR** area of $T = \left|\det(\boldsymbol{QP})^{-1}\right| \times$ area of T' ***(M1)***

$\Rightarrow$ area of $T = 273 \times \frac{1}{78}$

$= 3.5$ (cm^2) ***A1***

Note: Award ***(A1)(M0)A0*** for an answer of -3.5 (cm^2) with or without working. Accept an answer of 4.04 (cm^2) from use of 3sf values in their answer to part (a).

[3 marks]
Total [7 marks]

14. (a) $\boldsymbol{v}_B = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$ ***(A1)***

attempt to find any relevant angle ***(M1)***

$\tan^{-1}\left(\frac{3}{2}\right)$ $\left(= 56.3099...^\circ\right)$

$(90^\circ + 56.3099...^\circ =)$ 146° $(146.3099...^\circ)$ ***A1***

[3 marks]

(b) setting $1 + 2t = -2 + 4t$ ***(M1)***

$t = 1.5$ (hrs.) ***A1***

[2 marks]

(c) $\boldsymbol{r}_B - \boldsymbol{r}_A = (-3 + 2t)\boldsymbol{i} + (-7 + 4t)\boldsymbol{j}$ ***(M1)***

$-3 + 2t = -(-7 + 4t)$ ***(M1)***

$t = 1.67$ $\left(\text{hrs.}\right)$ $\left(1.66666..., \frac{5}{3}\right)$ ***A1***

[3 marks]

Total [8 marks]

15. (a) (i) 224 g (224.25 g) ***A1***

(ii) $\left[222.1,\ 226.4\right]$ ***A1A1***

Note: Award ***A1*** for each correct end of the interval. Accept open or closed (weak or strict) interval notation. Inequalities involving μ would also be accepted, but not involving $\bar{x}$.
Award ***A1A0*** for correct answers not given correct to 4 sf.

[3 marks]

(b) **EITHER**
the (population) weight of granules of Apollo coffee is normally distributed. ***R1***
OR
the readings are independent ***R1***

[1 mark]

(c) 226g lies within the confidence interval, ***R1***
so there is no evidence to dispute the claim on the label. ***A1***

Note: Do not award ***R0A1***.

[2 marks]

Total [6 marks]

16. (a) $(1.04,\ 0.509)$ $((1.03667...,0.509085...))$ ***A1A1***

[2 marks]

(b) attempt to make x the subject for either function ***(M1)***

$x = 4y^2$, $x = \cos^{-1} y$ ***A1A1***

attempt to use $V = \pi\int x^2 \mathrm{d}y$ ***(M1)***

$$V = \pi \int_0^{0.509085...} \left(4y^2\right)^2 \mathrm{d}y + \pi \int_{0.509085...}^{1} \left(\cos^{-1} y\right)^2 \mathrm{d}y$$ ***A1***

[5 marks]

(c) $= 1.15$ (units^3) ***A2***

Note: Do not ***FT*** from part (b) to part (c).
Award ***A1*** for 1.1. with no previous working.

[2 marks]
Total [9 marks]

17. (a) $\frac{\mathrm{d}y}{\mathrm{d}x}$ is undetermined at $(0, 1)$ ***R1***

$\left(\text{so cannot use } y_n = y_{n-1} + h\left(\frac{x}{(x^2+1)(2y-2)}\right)\right)$

Note: Accept "undefined", "indeterminate" or "division by zero" in place of "undetermined".

[1 mark]

(b) $\int (2y-2)\,\mathrm{d}y = \int \frac{x}{x^2+1}\mathrm{d}x$ ***M1***

$y^2 - 2y = \frac{1}{2}\ln(x^2+1) + c$ ***A1***

substituting $x = 0,\ y = 1$ ***M1***

$c = -1$

$y^2 - 2y + 1 = \frac{1}{2}\ln(x^2+1)$

$(y-1)^2 = \frac{1}{2}\ln(x^2+1)$ ***A1***

$y - 1 = \sqrt{\frac{1}{2}\ln(x^2+1)}$ (where positive root required as $y \geq 1$)

$y = 1 + \sqrt{\frac{\ln(x^2+1)}{2}}$ ***AG***

[4 marks]

(c) (when $x = 0.1$) $y = 1.07$ $(1.07053...)$ ***A1***

[1 mark]

Total [6 marks]